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Information Theoretic Models of HCI: A Comparison of the Hick-Hyman Law and Fitts' Law

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ABSTRACT

The Hick-Hyman Law and Fitts' Law are two surviving human performance principles based on Shannon and Weaver's (1949) Information Theory. In the early 1980s, Card, Moran, and Newell (1983) presented the laws as design principles for developers to maximize usability in the design of human–computer interfaces. A search of the current human–computer interaction (HCI) literature, however, will reveal that the Hick-Hyman Law failed to gain momentum in the field of HCI, whereas Fitts' Law received, and continues to receive, substantial attention. This article begins with a discussion the common information theoretical concepts of the two laws, and then examines each law with respect to its origins, theoretical formulation, theoretical development, research, and applications and examines the possible contributing factors responsible for the failure of Hick-Hyman Law to gain momentum in the field.

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CONTENTS

1. INTRODUCTION

Soon after C. E. Shannon published his seminal paper in *Bell System Technical Journal*, in which he introduced Communication Theory or more commonly, *Information Theory* (Shannon, 1948; Shannon & Weaver, 1949), psychologists immediately recognized that Information Theory might have significant relevance to psychology (Miller & Frick, 1949) and many began applying it to explain a variety of psychological problems (Attneave, 1959). This period in the 1950s begat what is known as *Information Processing Theory* through the efforts of Miller (1953, 1956), McGill (1954), and others (see Attneave, 1959; Garner, 1962). This approach of viewing humans as information processors has since matured and evolved to a point where the tenets of the approach bear little connection to information theoretic concepts. Luce (2003) writes:

> The word *information* has been almost seamlessly transformed into the concept of "information-processing models" in which information theory per se plays no role. The idea of the mind being an information-processing network with capacity limitations has stayed with us, but in far more complex ways than pure information theory. (p. 185–186)

Nevertheless, surviving psychological adaptations of the classical Information Theory were promptly applied to the fledgling field of human–computer interaction (HCI) in the early 1980s. In the first issue of *Human–Computer Interaction* journal, Newell and Card (1985) articulated the prospective role of psychology in HCI. Their overall vision depicted the application of psychological principles to the design of human–computer interface by developers (Card, Moran, & Newell, 1983). In one of the models put forth to guide developers, the Model Human Processor (MHP), Card et al. proposed two general psychological information theoretic models as two of the principles of operation: the *Hick-Hyman Law* (Hick, 1952; Hyman, 1953) for choice-reaction time and *Fitts' Law* (Fitts, 1954; Fitts & Peterson, 1964) for motor performance (Card et al., 1983, pp. 23–97).

A search through the leading HCI literature will reveal an interesting distinction between the two laws since Card et al. (1983). Despite their common theoretical roots, the laws have received different levels of acceptance in the field of HCI. Fitts' Law has enjoyed and continues to receive a great deal of attention in the field but the same cannot be said for the Hick-Hyman Law. This article reviews each law with respects to its origins, theoretical formulation, theoretical development, research, and application and discusses the possible contributing factors responsible for the failure of Hick-Hyman Law to gain momentum in the field.

2. PSYCHOLOGY AND HCI

To put the relationship between psychology and HCI into perspective, we expound on the vision of the role of psychology in HCI set by researchers 2 decades ago (Card et al., 1983; Newell & Card, 1985). The statement of the prospective role for psychology in the field was prefaced by the researchers' concern that in the fledgling field of HCI, hard science (computer science) would drive out soft science (psychology). The preventive solution they proposed was to "harden" psychology, that is, to improve the scientific caliber of the discipline to prevent its displacement. Another concern that Newell and Card (1985) expressed was that psychology might take a backseat and function merely as an evaluation tool. As such, Card et al. (1983) favored psychological application during the *design* phase over the *evaluation* phase in the design cycle of a human–computer interface.

It should be noted that Card and his colleagues advocated psychological applications to HCI problems but they did not envision psychologists as the primary professionals applying them. The researchers saw three possible roles for psychology: (a) Psychologists could be the primary professionals in HCI, as they are in some fields like mental health and counseling, (b) psychologists could be specialists working with the primary professionals, the system designers, and (c) last, the primary professionals—the system designers—could apply psychology themselves. Because the field was already dominated by computer scientists as primary professionals, and the authors felt that having psychologists function as specialists distances psychology from the design process, the researchers favored the last alternative where system designers would directly apply psychological knowledge.

It was in this momentum when the Hick-Hyman Law and Fitts' Law, along with others, were put forth as guidelines for developers. The laws were intended to highlight basic perceptual and psychomotor principles, so that developers could maximize usability of their products with these principles in mind. To appreciate the theoretical roots of these laws, we turn to a concise discussion of the relevant concepts of classical Information Theory.

3. INFORMATION THEORY

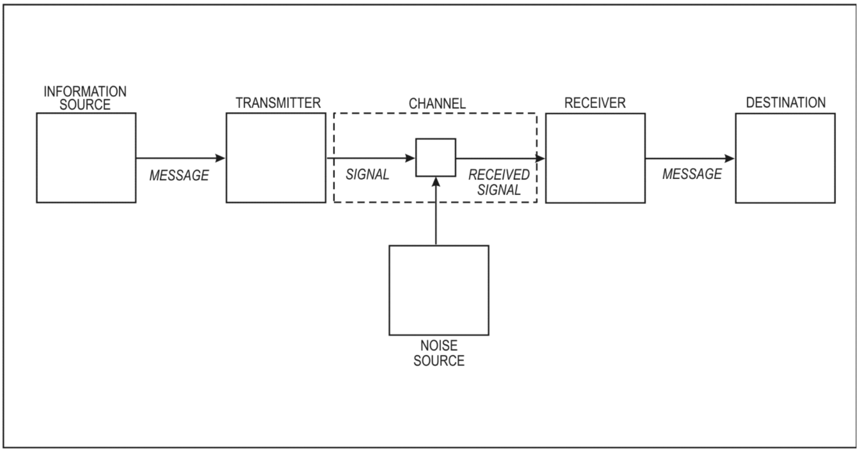
It is beyond the scope of this article to provide a thorough coverage of all the concepts of Information Theory as adapted by psychology. As such, this section presents only fundamental concepts that are pertinent to the two laws. Monographs and articles that provide accessible introduction to Information Theory as adapted by psychology include Hick (1952, 1953), Crossman (1953), Miller (1953, 1956), McGill (1954), and Attneave (1959).

3.1. The Communication System

The classical Information Theory is essentially a communication engineering theory based on the prior works of Nyquist (1924, as cited by Shannon, 1948) and Hartley (1928, as cited by Shannon, 1948) on the transmission of electrical signals for telegraphic communication. The model upon which Information Theory was formulated is specified in the following components (Shannon, 1948): The *information source* produces a message or sequences of message; the *transmitter* operates on the message to make it transmissible through a medium called the *channel*; a transmitted message reaches the *receiver* that reconstructs the message to the *destination* (See Figure 1). The Hick-Hyman Law and Fitts' Law are based on analogies of this general model of communication system.

The amount of information that a communication channel transmits in a fixed amount of time is referred to as the *channel capacity* (C). Channels are

Figure 1. **Schematic diagram of a general model of communication system (Shannon-Weaver Model). Adapted and modified from Shannon (1948, Figure 1, p. 381) with permission of Wiley Publishing Inc., a subsidiary of John Wiley & Sons, Inc.**

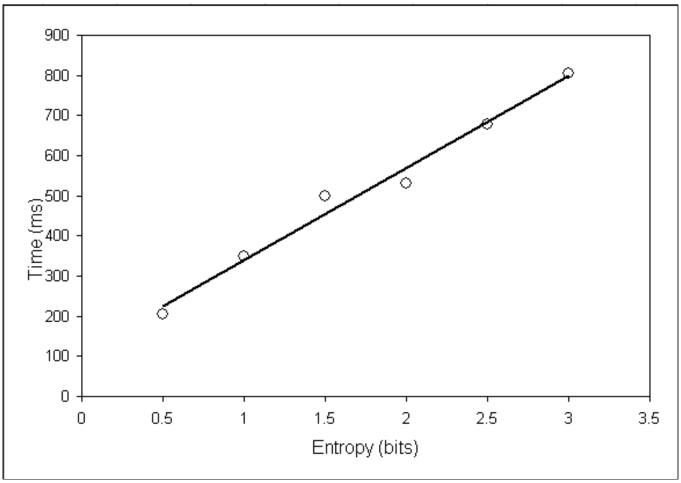


bound by physical limitations and thus have different capacities. An illustration is the distinction between browsing the Internet over telephone dial-up, which can transmit about 56 kilobits of data in a second (kbps), and over broadband that will transmit in excess of 20 times of the same amount in a second (1.5 Mbps). Needless to say, the difference between the two means of browsing the Internet is transmission speed. As a footnote, the speeds of 56 kbps and 1.5 Mbps are hardly reached because of attenuating factors, such as certain protocol overheads. As such, manufacturers of devices and service providers often put disclaimers to state that these are theoretical maximum speeds as determined by the hardware and other factors.

This leads to an important distinction between the classical Information Theory and the psychological ones based on it. Engineers can calculate the theoretical channel capacity by knowing the physical specification of the hardware (bandwidth, transmitter, type of cable, distance, etc.). Experimental psychologists have no a priori means of determining the channel capacity of a sensory, cognitive, or motor system. What they can do, however, is to measure information processing performance to infer the information capacity of the psychological system. Hick's (1952) and Hyman's (1953) experiments assessed the cognitive information capacity in choice-reaction experiments. Likewise, Fitts' (1954) work was an empirical determination of the information capacity of the human motor system.

The means by which Hick, Hyman, Fitts, and others determine information capacity is similar. As an illustration, Figure 2 shows six data points from

Figure 2. **Hypothetical data showing the concept of the reciprocal of the slope as information capacity. Values of the dependent measure, time (ms), are empirically determined. Values of the independent measure values, entropy, are calculated from the task as a measure of information potential.**



a hypothetical experiment. Suppose *m* is the slope of the regression, which serves an index of the time taken to process one bit of information. The reciprocal of *m* (when converted into seconds) represents the amount of bits that is processed in a second or the empirical information capacity (Laming, 1968; MacKenzie, 1992). Suppose *m* is 240 msec/bit, then m^{-1} is 0.00416 bits/msec or 4.16 bits/sec. This is the *index of performance* (IP) in Fitts (1954) and the *rate of gain of information* in Hick (1952).

3.2. Quantifying Information

Information is formally defined in Information Theory as a *reduction in uncertainty* (Shannon & Weaver, 1949) and quantified in units of *bit.* (Appendix A provides a primer on how entropy is conceptualized in Information Theory.) The Shannon-Weiner measure of information,

$$H = \sum_{i=1}^{n} p_i \log_2 \left(\frac{1}{p_i} \right) \tag{1}$$

commonly expressed as

$$H = \sum_{i=1}^{n} p_i \log_2 p_i \tag{2}$$

where n is the number of alternatives, and p_i is the probability of the ith alternative, yields the entropy, H, of a message (or events, signal, stimuli, etc.) that is to be transmitted. In psychology, this entropy (or expected information potential) of a stimulus or a set of stimuli is frequently designated H_{av} or H_{ave} (for average information) when the alternatives are not equiprobable, and H_{max} (for maximum information) when the alternatives are equiprobable. In the former, when alternatives bear different probabilities, the entropy of the stimulus or set of stimuli will be compromised (reduced). In the latter, the entropy will be maximal. Hyman (1953) exploited this effect of nonequiprobability on the entropy of a set of visual stimuli to yield varying degrees of entropies that are less than the maximal entropy.

So far, H represents the *expected* information of the source and is designated $H(x)$. What is actually *received* at the destination is designated $H(y)$. Because there is usually interference during transmission, the average information, R (or designated H_T in Hick's paradigm), that was faithfully transmitted is calculated as:

$$R = H(x) - H_y(x) \tag{3}$$

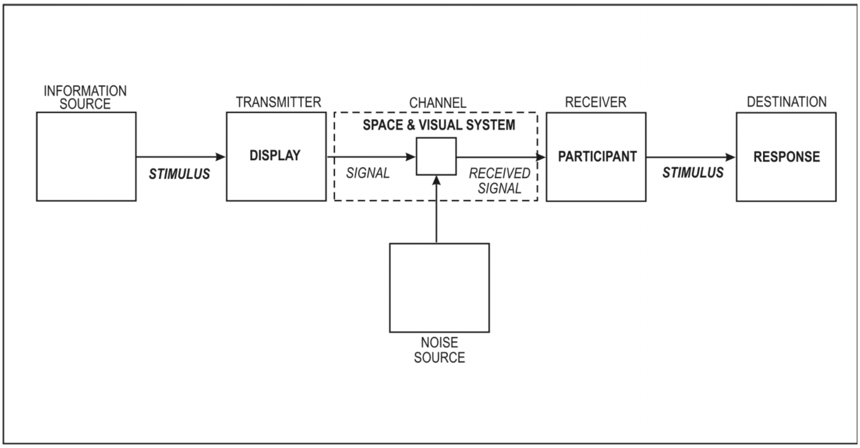
where $H_y(x)$ is the *equivocation* or the conditional entropy of x when y is known. In Hick's and Fitts' paradigms, when a participant performs a task without errors, he is said to be extracting all the expected information of the stimuli. As such, $H_y(x)$ equals zero. When errors are made, which is often the case, the experimenter has to determine $H_y(x)$ to report the actual average information transmitted.

It must be noted that although Information Theory provided a "yardstick" or metric to quantify the information content of a stimulus or a set of stimuli for experimental psychologists (Miller, 1953), not all endeavors to apply Information Theory to psychological problems were successful. Attneave (1959) described that "some of these attempts were successful and illuminating, some were pointless, and some were downright bizarre" (p. v). There is reason to believe that even Shannon himself was skeptical of some of the work that stemmed from his theory, stating that "Information Theory has perhaps ballooned to an importance beyond its actual accomplishment" (as cited by Johnson, 2001). Luce (2003) recently referred to Information Theory as a *fad* in psychology in the 1950s and 1960s. Others have highlighted the fact that the concepts in Information Theory are descriptive and not explanatory (Baird, 1984; Sanders & McCormick, 1987).

4. THE HICK-HYMAN LAW

The Hick-Hyman Law (Hick, 1952; Hyman, 1953) was built upon prior findings of a systematic relationship between number of alternate stimuli and

Figure 3. **Schematic diagram of the choice-reaction time experiment as a model of a communication system. Adapted from Shannon (1948) and information from Laming (1968) with permission of Wiley Publishing Inc., a subsidiary of John Wiley & Sons, Inc.**



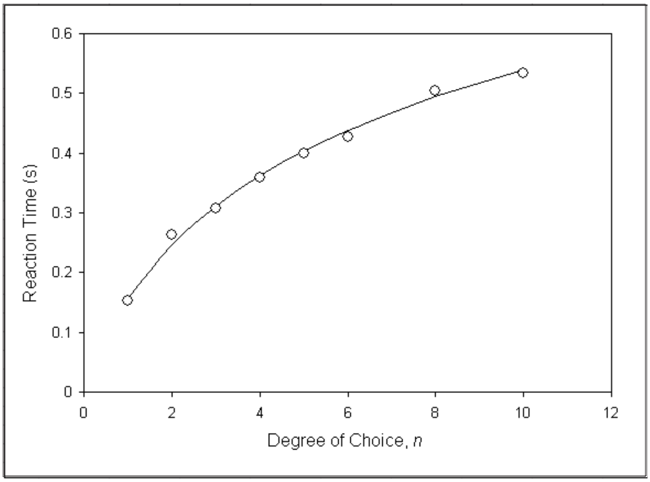
choice-reaction times. This was first reported by F. C. Donders (1868, as cited by Broadbent & Gregory, 1962) and later by J. Merkel (1885; cited by Hick, 1952). Merkel discovered that it takes longer to respond to a stimulus when it belongs to a large set as opposed to a smaller set of stimuli. This regularity caught the attention of psychologists who saw its analogy to the classic Information Theory: The display is the *transmitter* of information; each alternate stimulus the *message*; the sensory-perceptual system the *channel*; the participant the *receiver*, and the appropriate action the *destination* (Hyman, 1953; Laming, 1968; see Figure 3).

4.1. Hick (1952) Original Experiments

Hick was probably the first to apply Information Theory to psychological problems (Hick, 1953). He used 10 pea lamps arranged in an irregular circle formation and connected them to a device that was (punch-tape) coded to light one random lamp every 5 sec (Hick, 1951). The response manipulandum was a set of 10 corresponding Morse keys, one for each of the participant's fingers. The participant's task was to depress the correct key for a lighting of a particular lamp. Both stimulus presentation and response were recorded in binary code by moving paper. (An author's impression of Hick's apparatus is provided in Appendix B to appreciate the psychophysical instruments used by experimental psychologists of his days.)

The goal of the first experiment was to determine the empirical relationship between choice reaction time and stimulus information content (en-

Figure 4. **Data of subject A (Hick himself) in experiment I in Hick (1952). Adapted from Hick (1952, Figure 1, p. 15) using DataThief II (Tummers, 2000) with permission of Psychology Press, http://www.psypress.co.uk/journals.asp. The data are fitted with a logarithm function of 0.518 $\log_{10}$ (n + 1) from Hick (1952).**



tropy). Serving as the sole participant, Hick performed the task using a range of 2 to 10 alternative stimuli, and imposed upon himself to attain errorless responses. His results, based on over 2,400 responses, are shown in Figure 4.

In Experiment II, Hick first trained his participant on a task with 10 alternatives. The participant then continued to perform the task in three phases. The participant was initially encouraged to perform as fast as possible, then he was instructed to perform as accurately as possible, and finally he was instructed to perform as fast as possible again in the last phase. The eight data points (circles) located at the top right quadrant in Figure 5 represents the data that were produced during the training. The data points marked as diamonds represent reaction times (RTs) of the trials when the participant was encouraged to be fast. Recall that Experiment II employed 10 stimuli, not a range of stimulus set sizes. As such, the abscissa represents a second dependent value expressed as any positive real number called the equivalent degree of choice, n_e. This is calculated by taking the participant's errors into consideration (Equation 3) such that "if there were no mistakes it would mean that all the information was being extracted, and n_e would be 10" (Hick, 1952, p. 15). Hick called the n_e "the antilogarithm of the information gained" (p. 16), because he calculated the alternatives from the information gained, instead of using the alternatives to compute the entropy. As such, we can infer then that the first data point on the left (diamond) in Figure 6 would represent a trial in which many response errors occurred, which resulted

Figure 5. **Data of subject B in experiments II and III in Hick (1952). Adapted from Hick (1952, Figure 2, p. 17) using DataThief II (Tummers, 2000). Data of the later runs of experiment II are fitted with a logarithmic function of $RT = -0.042 + 0.519 \log10(n_e + 1)$ from Hick (1952).**

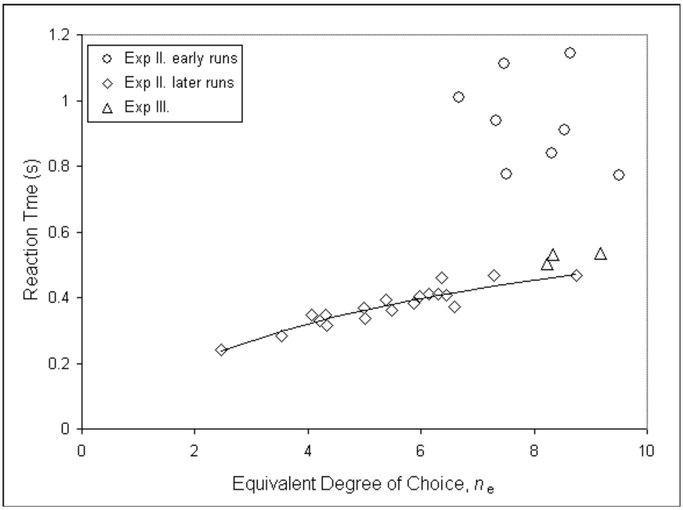
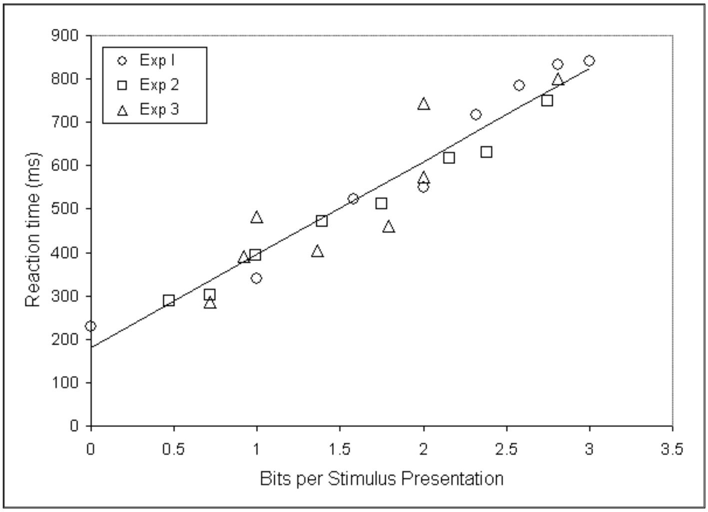


Figure 6. **Data of subject FP in Hyman (1953). Adapted from Hyman (1953, p. 192, figure 1, left-bottom panel) with dependent measures estimated from DataThief II (Tummers, 2000). The data points of all three experiments are fitted with a linear function of 180 + $215H_T$, where H_T is the bits per stimulus presentation (Hick, 1953).**



in a low quantity of information transmitted. In this case, it is calculated to be "worth" a little over 1 bit or equivalent to a little above 2 choices. (Hick reported that error rates were as high as 70%.)

Results of Experiment II showed that even when a participant was instructed to perform rapidly and to make "as many errors as he liked" (p. 16), the function of these data points superpose the function established in Experiment I with "errorless" data points. This demonstrated that the RT was a logarithmic function of average information transmitted, H_T, regardless of whether H_T was the residual entropy of the stimuli after equivocation ($H_y[x] > 0$) in Experiment II or the maximal entropy available from varying number of alternatives without equivocation ($H_y[x] = 0$) in Experiment I.

To ensure that the performance of his participant was not due to learning of a specific set of stimuli, Hick conducted Experiment III with a new set of stimuli. Results show that the mean RT of the new set of stimuli fell along the previous empirical logarithmic function (Figure 5, triangles), which suggested that the effects of learning were negligible in the experiment.

4.2. Hyman (1953) Original Experiments

Hick (1952) characterized the relationships between RT and n or n_e as logarithmic and concluded that "the amount of information extracted is proportional to the time taken to extract it, on the average" (p. 25). He did not, however, explicitly postulate a linear relationship between RT and H_T. Note that the data Hick presented were plotted as a function of the number of alternatives (n or n_e). Under the supervision of Hick, Crossman (1953), using a card-sorting task, presented data that were plotted as a function of H_T. Hyman (1953) may be the first to articulate the linearity between the two variables:

> So far this paper has presented empirical relationships which suggest that reaction time can be considered a linear function of stimulus information within the range of 0.00 to 3.00 bits. (p. 193)

Although Hick has demonstrated that one can reduce entropy by reducing the number of alternatives or by factoring out equivocation, Hyman (1953), as well as Crossman (1953), employed a third method to reduce the entropy of a set of stimulus. Exploiting the fact that entropy is maximal when the stimuli are equiprobable, Hyman altered the probabilities of the stimuli (such that they are not equiprobable) to yield varying amounts of entropy so that he can assess RT as a function of H_T.

Hyman used 8 lights in a matrix of 36 lights (6 rows by 6 columns) display and designated names—*Bun*, *Boo*, *Bee*, *Bore*, *By*, *Bix*, *Bev*, and *Bate*—to each of them. At the beginning of each trial, the experiment first gave a warning signal and 2 sec later turned on one of the eight lights and started a timer simultaneously. Participants responded by calling out the designated name of the light. A throat microphone attached to the participant activated an electronic voice key to stop the timer. (An author's impression of Hyman's apparatus is provided in Appendix C.)

The first experiment replicated the procedure of Hick (1952) in that sets of alternative stimuli, ranging from one to eight in size, were presented at equal probabilities. These sets of stimuli yield bits ranging from 0 to 3. Hyman (1953) replicated the results of Merkel (1885) and Hick (1952) with voice keys in this experiment. Experiment II comprised eight conditions, each with differing set sizes and probabilities for each alternative, which collectively yielded bits ranging from 0.47 to 2.75. The last experiment also had eight conditions. In each condition, each of the alternatives had equal likelihood of occurring but its probability is conditional. For example, in condition 1, where two alternatives were used, the conditional probability of b given that a has occurred or $p(b|a)$ is 0.8. These conditions yielded bits ranging from 0.72 to 2.81. Hyman found that RT was linear as a function of bits of the alternatives with unequal probabilities, suggesting that RT was indeed a function of stimulus information (entropy) and not merely a function of the number of alternatives. Results of Experiment III were slightly different, but similar enough to the findings of Experiment II, which confirmed Hyman's hypothesis that all results will yield identical regression lines (Figure 6).

With the extension of Hyman (1953), Hick's Law was consequently accepted by many as the Hick-Hyman Law. Essentially, the law predicts a linear relationship between reaction time and transmitted information:

$$RT = a + bH_T \tag{4}$$

where RT is reaction time, a and b are empirically determined constants, and H_T is the transmitted information. The reciprocal of b is what Hick referred to as the rate of gain of information or the information capacity.

4.3. Theoretical Developments

The extent to which RT is linear to H_T has received a lot of attention. For example, Longstreth, El-Zahhar, and Alcorn (1985) reported that there is little increase of RT when the stimuli are familiar letters or digits and when the responses are verbal identification. They noted that their data formed a downward inflected curve not an uninflected line, which Hick's Law could not fit perfectly. Longstreth and his colleagues (1985) wrote, "any such theory is in doubt because, in our opinion, the law is false" (p. 431). They submitted a power curve as a replacement for fitting the data:

$$RT = a + b(1 - N^{-1}) \tag{5}$$

In response to Longstreth et al.'s model, Welford (1987) pointed out that when the model is applied to certain data sets, negative intercepts are derived, which is illogical for the dependent measure of time, and that the equation predicts a decreasing RT variability as a function of the number of alternatives, when empirically, the opposite has been observed. Longstreth and Alcorn (1987) offered their rebuttal but Longstreth (1988) may have relented when he conceded that the linearity as predicted by the law is true but is bounded by an upper limit of 3 bits.

Hick originally described the process underlying the relationship between the number of choices and reaction time as a sequential and hierarchical process (Hick, 1952) but this was challenged by Laming (1968). Parallel exhaustive process models were suggested as replacement of Hick's serial process models (Christie & Luce, 1956; Laming, 1966). There are recent efforts in trying to understand the underlying process as captured by the law. One recent process model is Usher and McClelland's (2001) Leaky, Competing Accumulator Model. The model is built upon stochastic information accumulation models (see Townsend and Ashby, 1983) that share two principles of information processing: (a) information is accrued in a gradual process and (b) the accumulated information is subject to random fluctuations (Usher & McClelland, 2001). Usher and McClelland's model incorporated a decay or leakage process and a competition process between representations of alternative outcomes into a stochastic model and they reported that their model was able to explain the regularity captured by the law.

4.4. Research and Applications

Hick (1953) hoped that his work would continue to include "different types of control and display codes" (p. 133) and some have taken up the challenge. For example, other quantifiable aspects of the stimulus have been used to vary the information content in a Hick's paradigm. Levy and Norton (1972) investigated the dimensionality of the stimulus. Varying the size and brightness of visual stimuli to yield different amounts of information, the investigators found that CRT was linear as a function of bits as held by the law. The following summarizes the more recent areas of research that involved the use of the Hick-Hyman Law.

Speed-Accuracy Tradeoff

An area that is frequently investigated with the Hick-Hyman Law is *speed-accuracy tradeoff.* It has been commonly known that in conditions where participants are instructed to speed up performance, faster responses are produced at the expense of accuracy. The converse is true; when participants are instructed to be as accurate as possible, speed of performance is compromised (Pachella & Pew, 1968). Hick (1952) demonstrated this phenomenon in Experiment II when he had his participant perform as accurately as possible during the early runs. Data points of these early runs are loosely clustered at the top right quadrant (Figure 5, circles), indicating that participants are accurate at the expense of speed. When the participant was instructed to perform as quickly as possible, data points were aligned along what Hick described as a logarithmic function (Figure 5, diamonds), indicating that they were performing at maximal speeds but accuracy was compromised. The information theoretic relation to the speed-accuracy tradeoff is demonstrated by comparing the amount of information processed in both runs. More information was processed in the "accuracy" run than the "speed" run. Using payoff and feedback appears to be effective in motivating participants to focus on speed or accuracy (Fitts, 1966). The speed-accuracy phenomenon has also been investigated in stimulus categorization (Swanson & Briggs, 1969) and absolute judgment of visual stimuli (Pachella & Fisher, 1972).

Stimulus-Response Compatibility

Stimulus-Response (S-R) compatibility (SRC) refers to the degree of compatibility between the presentation of a stimulus and the means of responding (Fitts & Deininger, 1954; Fitts & Seeger, 1954). Compatible S-R pairs facilitate the responding of a stimulus, thus yielding a higher rate of information transfer (Fitts & Seeger, 1953), whereas incompatible ones impede optimal performance. A classic application example is the use of a wheel-like knob in a cockpit for the lever that controls the wheels.[1] Several investigators have reported demonstrable SRC effects on the slope parameter of the Hick-Hyman Law. Specifically, an increase in SRC has been found to diminish the slope of the RT points as a function of the number of alternatives (Brainard, Irby, Fitts, &

1. During World War II, psychologists were employed to assist the military in selection and training of personnel to reduce manmade errors (Grether, 1968, 1995; Nickerson, 1999; Roscoe, 1997). Roscoe (1997) described how pilots of certain military aircrafts would retract the wheels instead of the flaps of the aircraft after landing, causing the aircraft to smash to the ground. Lt. Alphonse Chapanis, a psychologist, noticed that the wheel and flap controls were nearly identical and positioned side by side, and determined that the pilot errors were really cockpit design errors. His solution was to attach a rubber wheel-like knob to the wheel control and a wedge-shaped knob to the flap control. This solution was so effective that it can still be found in the cockpits of modern commercial aircrafts like the Boeing 747. Fitts (1951) presented a host of other pilot errors that resulted from responding to instruments and signals, and presented a voluminous collection of empirical data on equipment design.

Alluisi, 1962; Broadbent & Gregory, 1962, 1965; Davis, Moray, & Treisman, 1961; Mowbray & Rhoades, 1959; Pierce & Karlin, 1957).

Psychometrics

Many current research studies on the Hick-Hyman Law can be found in psychometrics. Roth (1964; cited by Jensen & Munro, 1979) is commonly cited as one of the first to investigate the *RT-IQ* relationship, but the interest in reaction time and intelligence can be dated back to F. Galton (1883). Presently, A. R. Jensen is a leading proponent of research into the relationship between RT and intelligence (IQ) (Nettlebeck, 1998). Empirical parameters of the intercept (*a*), slope (*b*), and RT (*M* and *SD*) (see Equation 7) have all been correlated against intelligence (Roth, 1964; cited by Jensen & Munro, 1979; Smith, 1989; Neubauer, 1990, 1991). The *a* parameter reflects individual differences in sensory-motor lags in task performance and has been found to have a higher negative correlation with IQ than *b* (Jensen, 1998). Jensen and Munro (1979) reported a mean correlation of –.41 between Raven's Standard Progressive Matrices, a 60-item intelligence test, and RT. An intervening factor that has been found to modulate the strength of correlation between RT and IQ is SRC (Fitts & Seeger, 1954). Neubauer (1991) manipulated the SRC into high (easy) and low (difficult) groups, where the latter would require participants to use different rules for each trial to map signals to responses and found a relatively higher RT-IQ correlation in the low S-R group. (See Smith, 1989, for a meta-analysis of seven studies.)

HCI Applications

As previously mentioned, applications of Hick-Hyman Law are scarce in the HCI literature despite its foundation in information processing. Card et al. (1983) introduced the Uncertainty Principle by presenting Hyman's (1953) data and a hypothetical scenario of a telephone call director to demonstrate the application of the principle (Card et al., 1983, p. 74). Olson and Nilsen (1988) compared the decision time taken to perform equivalent functions in two spreadsheet software (Lotus 1–2–3 and Multiplan). In the task defined in the experiment, Lotus 1–2–3 had three methods available to users to perform a particular task and Multiplan only had one to perform the similar task. The investigators found that users took additional time to decide which of the three alternates to use in Lotus 1–2–3. Landauer and Nachbar (1985) used the Hick-Hyman Law in their study on response time in menu selection using touchscreen. They reported that the results adhered to the Hick-Hyman Law but was unable to proffer any practical menu design: "more results from experiments like these … will clearly be needed before more confident generalization to new cases becomes feasible" (p. 77). Examples of other HCI applications, such as soft keyboards (Soukoreff & MacKenzie, 1995), of Hick-Hyman's Law incorporate Fitt's Law. This shall be elaborated in a later section.

5. FITTS' LAW

Fitts' Law (Fitts, 1954; Fitts & Peterson, 1964) is the other surviving Information Theory model in psychology (MacKenzie, 1992). Essentially, the law states a linear relationship between task difficulty and movement time (MT). Fitts quantified and expressed task difficulty as an *index of difficulty* (ID), which is specified by the distance between two stationary targets that needs to be covered called the *amplitude* of the movement (A) and the *width* (W) of the targets where the movement must terminate:

$$ID = \log_2\left(\frac{2A}{W}\right) \tag{6}$$

Adopting Shannon's (1948) Theorem 17, Fitts conceptualized the human motor system as a communication *channel*, movement amplitude as the *signal*, and target width as the *noise* (MacKenzie, 1989; Figure 7). By combining various degrees of A and W, Fitts was able to vary the information content of ID (measured in bits) and determine the *information capacity* of the human motor system in controlling amplitude of movement.

5.1. Fitts (1954) Original Experiments

Fitts ran four experiments using the reciprocal tapping, disk transfer, and pin transfer tasks. In the reciprocal tapping task in Experiment I, participants used a metal-tipped stylus (1 oz. version on the first day; 1 lb. version on the second day) to tap two stationary strips of metallic targets. The width of the plates (W) varied from .25 to 2 in., and the distance between them varied from 2 to 16 in. Participants were instructed to strike the target places alternately to score as many hits as possible. In other words, accuracy was encouraged (Fitts, 1954, p. 384). In the disk transfer task in Experiment II, participants were instructed to transfer and stack round plastic discs (with holes drilled through the middle) from one pin to another. Holes of different sizes and pins of different diameters were used. In the pin transfer task in Experiment III, participants were instructed to transfer pins of different diameters from one set of holes to another set of holes.

The results of all the experiments in Fitts (1954) are consolidated in Figure 8. In Experiment I, Fitts found the average error rates to be negligible be-

Figure 7. **Schematic diagram of human motor system experiments as a model of communication system (Fitts, 1954; MacKenzie, 1989). Adapted and modified from Shannon (1948, Figure 1, p. 381) with permission of Wiley Publishing Inc., a subsidiary of John Wiley & Sons, Inc.**

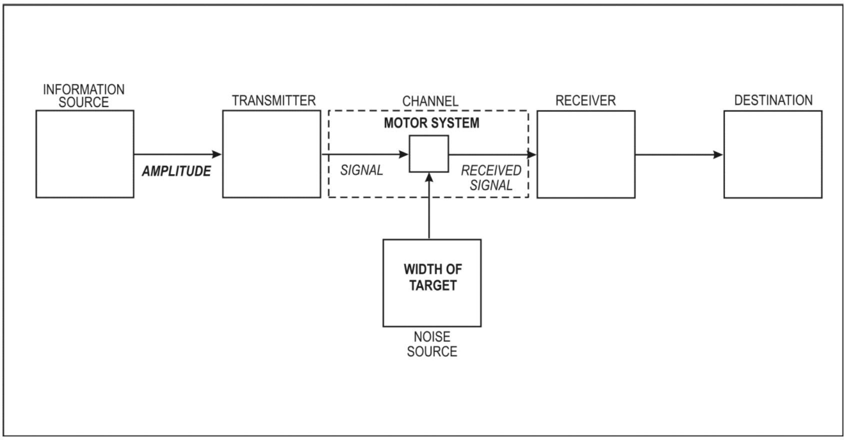
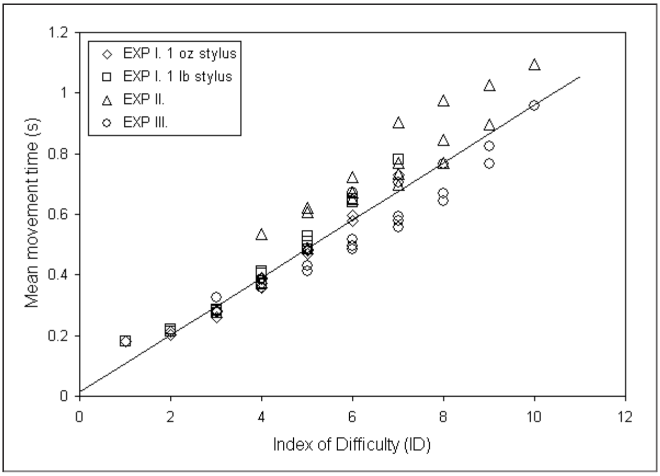


Figure 8. **Results of Fitts' (1954) experiment I (p. 385, Table 1), experiment II (p. 386, Table 2), and experiment III (p. 388, Table 3). The function of the linear fit, *MT* = 12.8 + 94.7 *ID*, was calculated by other investigators (MacKenzie, 1992).**



tween the two styluses and that the most difficult condition was the condition with the smallest W and the largest A, which yielded error rates of 3.6% with the lighter stylus and 4.1% with the heavier one. Overall, for each length of the target width, W, movement time, MT, increased along with the distance between the targets, A. Fitts did not elaborate on the results of Experiments II and III but plotting the tabulated results (Fitts, 1954, Table 2, p. 386, & Table 3, p. 388) show that they are consistent with those of Experiment I: When the ID increased, MT increased linearly.

Fitts reported an index of performance (IP) from his results. In Fitts' paradigm, this index shows the capacity of the human motor system. A high IP translates to a large capacity. The index is calculated by dividing ID (Equation 7) by the empirically determined movement time, MT:

$$IP = \frac{ID}{MT} \tag{7}$$

This *index of performance,* or commonly called *throughput* (TP) by some HCI researchers (MacKenzie and Soukoreff, 2003; Zhai, 2002), is measured in bits per unit time and is homologous to the rate of gain of information in Hick's (1952) paradigm and analogous to the channel capacity in Shannon and Weaver's (1949) theory.

The confirmation of Fitts' hypothesis was shown by the constancy of IP over the combinations of A and W. Fitts reported that IP ranged from 10.3 to 11.5 bits/s in Experiment I; 7.5 to 10.4 bits/sec in Experiment II; and 8.9 to 12.6 bits/sec in Experiment III. It should be noted that these values that represent the maximum information capacity are dependent on various factors. For example, Crossman (1960) extended Fitts' Law to pursuit tracking and reported the information capacity of the task at 4 bits/sec with continual improvements up to 8 bits/sec.

Fitts' Law is traditionally expressed as:

$$MT = a + b\log_2\left(\frac{2A}{W}\right) \tag{8}$$

where a (intercept) and b (slope) are empirically determined non-negative constants. The multiplication of 2 to A in Equation 8 was arbitrary to prevent the logarithm from becoming negative (Bainbridge & Sanders, 1972; Welford, 1960). Equation 8 for Fitts' data have been calculated by others as (MacKenzie, 1992) as MT = 12.8 + 94.7 ID.

Equation 8 was derived from Shannon's Theorem 17 (Shannon, 1948), which recognizes that noise compromises the capacity of a channel to transmit information. Theorem 17 expresses the effective *channel capacity* (C) as:

$$C = W\log_2\left(\frac{P+N}{N}\right) \tag{9}$$

where W is the bandwidth (different from the W in Fitts' Law and sometimes designated B), P is the signal power and N is the noise power (Shannon, 1948, p. 639–642).

5.2. Theoretical Developments

There are significant theoretical modifications to the original Fitts' equation (Equation 8). A widely adopted modification is by Welford (1960). One will notice that the numerators and denominators of the multiplicand of the logarithmic term in Equations 8 (Fitts' Law) and 9 (Theorem 17) differ in placement, considering Fitts' analogy of A (amplitude) as P (signal power) and W (width) as N (noise) (MacKenzie, 1989). Welford reformulated Equation 8 to closer resemble Theorem 17:

$$MT = a + b\log_2\left(\frac{A + 0.5W}{W}\right) \tag{10}$$

Many, including Fitts (Fitts & Peterson, 1964) himself, have reported a better correlation between MT and ID using Equation 10 (MacKenzie, 1992; Roberts, 1997), although others found support for the original equation (Bainbridge & Sanders, 1972). Noting that Fitts' original equation departs unnecessarily from Shannon's, MacKenzie (1992) proposed another modification that directly resembles the latter:

$$MT = a + b\log_2\left(\frac{A + W}{W}\right) \tag{11}$$

MacKenzie replotted Fitts' (1954) data with adjusted W, and was able to produce a better fit of the data with Equation 11. Another popular modification is that of Meyer, Abrams, Kornblum, Wright, and Smith (1988). Meyer et al.'s model expresses Fitts' Law as:

$$MT = a + b\sqrt{\frac{A}{W}} \tag{12}$$

Other theoretical developments have focused on the calculations of the critical components in Fitts' Law. The calculation of IP, or more commonly referred to today in HCI as TP for throughput, as shown in Equation 7 can be shown to be the reciprocal of the slope parameter, *b*. This becomes problematic when one interprets this as the true capacity because the intercept, *a*, is not taken into account. A solution, as recommended by the ISO 9214–9, was to reduce both *a* and *b* into a single metric. Zhai (2002) presented the limitations of such an approach and proposed that both be reported as separate dependent measures. Additionally, Zhai (2002) advocated that researchers report both adjusted and unadjusted results with errors. The adjustment refers to the incorporation of error into the calculation of *MT*. This is done by calculating *ID* with an adjusted *W* that captures 96% of the hits in a task (see Figure

5 of MacKenzie, 1992). This adjustment, based on Fitts' (1954) observation that participants made approximately 4% errors, either reduces or increases the actual target width, W, into the effective width, W_e (MacKenzie, 1992).

MacKenzie (1992) stated that "despite being robust and highly replicable, Fitts' law remains an analogy waiting for a theory" (p. 100). Indeed, Fitts' Law is but a description of empirical data. At least two prominent models have been proposed to explain the processes that it captures—*deterministic iterative-corrections model* and s*tochastic optimized-submovement model.* The deterministic iterative-corrections model (Crossman & Goodeve, 1983) divides a single entire rapid-aimed movement into a series of submovements, each having its own home-to-target trajectory, which collectively determines the overall trajectory of the movement. The stochastic optimized-submovement model (Meyer et al., 1988, Meyer, Smith, Kornblum, Abrams, & Wright, 1990) divides a single entire rapid-aimed movement into two submovements: a primary submovement that forms the majority of the trajectory and an optional secondary submovement that, cued by visual or other feedback, functions to correct any error made by the first. In this model, an extension of Meyer et al. (1998), the ID is calculated by

$$ID = \left(\frac{A}{W}\right)^{\frac{1}{n}} \tag{13}$$

where *n* is number of submovements participants are assumed to need to traverse from home to target (Meyer et al., 1990). The errors are assumed to stem from neuromotor noise in the neuromotor system. Others have proposed a more kinematically plausible model based on the idea of neuromotor noise (Van Galen & De Jong, 1995).

5.3. Research and Applications

Most research outside of HCI can be arbitrarily divided into those with a kinematics focus and those with a neurocognitive focus. In the former, for instance, Fitts' Law has been applied to study movements made by the foot (Drury, 1975; Hoffmann, 1991), head (Jagacinksi & Monk, 1985), and different limbs (Langolf, Chaffin, & Foulke, 1976). Some have modified procedures within the Fitts' paradigm to determine the effects of the probe (Baird, Hoffmann, & Drury, 2002), to test pointing from two-dimensional (MacKenzie & Buxton, 1992) to three-dimensional tasks (Murata & Iwase, 2001), to assess performance of continuous, cyclical movements as opposed to discrete, alternating movements (Guiard, 1997), and to assess performance under water (Kerr, 1973). Although Fitts' Law appears to be more related to overt motor performance, it has relevance in the neurocognitive domains as well. Recently, many investigators have probed the neurocognitive substrates of motor imagery (Cerritelli, Maruff, Wilson, & Currie, 2000; Decety, 1996; Decety & Jeannerod, 1996; Maruff, Wilson, Fazio, Cerritelli, Hedt, & Currie, 1999). Results from these studies suggest that Fitts' Law holds true without the overt motor movements.

Speed-Accuracy Tradeoff

A common area where Fitts' Law and the Hick-Hyman Law converge is research on speed-accuracy tradeoff. The premise of Fitts' work rested on analyzing the variability of his participants' performance. Fitts wrote: "[t]he information capacity of the motor system, therefore, can be inferred from measures of the variability of successive responses that S attempts to make uniform" (Fitts, 1954, p. 382). The two metallic target strips of his reciprocal tapping apparatus were sandwiched between two "error plates" that were also wired to record overshoots and undershoots (Fitts, 1954, p. 384, Figure 1). Fitts assumed that the motor system has a fixed information capacity and that making participants perform beyond this capacity will result in systematic variability in responses: "if repetitive movements of a fixed amplitude is speeded up … movement variability will increase by a specific amount" (Fitts, 1954, p. 383). Fitts' assertion of an inverse correlation between the speed and accuracy is not uncontested. Schmidt, Zelaznik, Hawkins, Frank, and Quinn (1979) contended that the error made by participants in a Fitts paradigm is "linearly and directly related to movement amplitude, independent of movement time" (p. 446). Howarth, Beggs, and Bowden (1971) had also arrived at a similar conclusion.

Psychometrics

Like the Hick-Hyman Law, Fitts' Law has also attracted some attention in the psychometrics field. In Jensen and Munro's (1979) study, for instance, participants had to keep their finger on a "home" button and, upon the presentation of one of many green lights, lift their finger and hit a corresponding button to turn off the green light. The time taken to lift the finger from the home button was recorded as the choice-reaction time, CRT, and the time taken to go from the home button to the target was recorded as the movement time, MT. The investigators reported a mean correlation of −.46 between the Raven IQ scores and MT. This is not a conclusion accepted by others. In a large study, Roberts (1997) tested 179 participants in a Fitts' paradigm and administered a battery of 25 psychometric tests to the participants. He reported that the data conformed to Welford's (1960) variation of Fitts' Law but found no evidence for a correlation between IQ and MT. Roberts warned that "studies reporting significant correlation between *MT* parameters and intelligence should be viewed with suspicion" (p. 242).

HCI Applications

MacKenzie (1992) summarized six studies that applied Fitts' Law to HCI prior to 1992 (Card, English, & Burr, 1978; Drury, 1975; Epps, 1986; Jagacinksi & Monk, 1985; Kantowitz & Elvers, 1988; Ware & Mikaelian, 1987; as cited by Mackenzie, 1992). Chief among them is Card, English, and Burr (1978), regarded as the first application of Fitts' Law in HCI (MacKenzie & Soukoreff, 2003). Since Card et al. (1978), there are other works applying Fitts' Law to the use of pointing devices but only selected topics will be mentioned here.

Pointing. Fitts' Law has been applied to evaluate a variety of pointing devices. Kabbash, MacKenzie, and Buxton (1993), for example, compared the use of a mouse, a trackball, and a tablet-with-stylus in pointing and dragging tasks. The majority of the HCI research involves the physical operation (pointing, dragging, etc.) of a mouse or stylus to acquire a visual target on the screen. Gillan, Holden, Adam, Rudisill, and Magee (1990) extended Card et al.'s (1978) work by testing participants not only on a point-click task but also a point-drag task. In Card et al. and Gillan et al.'s (1990) paradigm, the task of point-click refers to moving the cursor from a starting point to a target (a word) in a block of text and clicking the target to select the target or the line, paragraph, or block of text where the target resides. The point-drag task refers to moving the cursor to a point in the text block, holding down a selection button or key while "dragging" the cursor to cover a desired portion of the text. In one experiment, Gillan et al. (1990) compared point-click and point-drag performance and found that pointing time in point-dragging was not related to the width of the text to be selected but was affected by the height of the text and distance of the text from the starting point. The researchers found that point-clicking was relatively faster and was sensitive to the width and height of the text and its distance from the starting point.

Angle of Approach. The angle of approach of the cursor has also been a focus of research by more than one group (MacKenzie & Buxton, 1992; Whisenand & Emurian, 1996). The original Fitts' paradigm is essentially a single dimensional (alternating left and right movement) task using vertically oriented rectangular targets. The heights were negligible and Fitts originally set the height at 6 inches but it was never factored as an independent measure. Accot and Zhai (2003) described the classical paradigm as AP or pointing with amplitude constraints and a paradigm with height constraints a DP or

Figure 9. **A hypothetical scenario where one is to find a correct symbol amoung four equiprodable alternatives. When no information is given at stage A, the amount of uncertainty is a bit. With each piece of information at stages B and C, the amount of alternatives is halved, thus reducing uncertainty until one knows what the correct symbol is.**

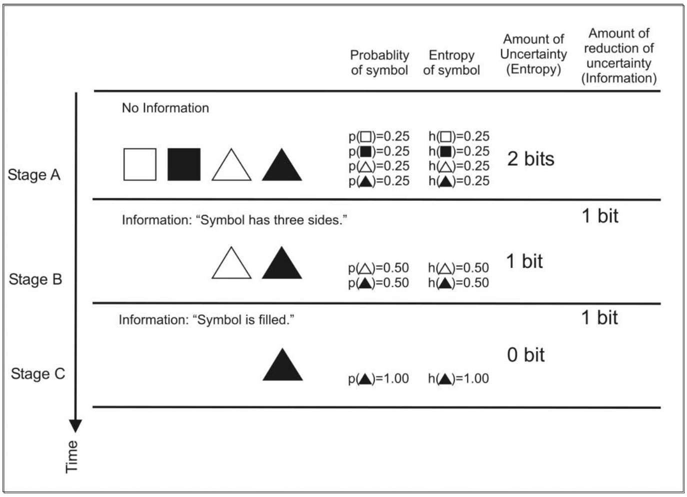
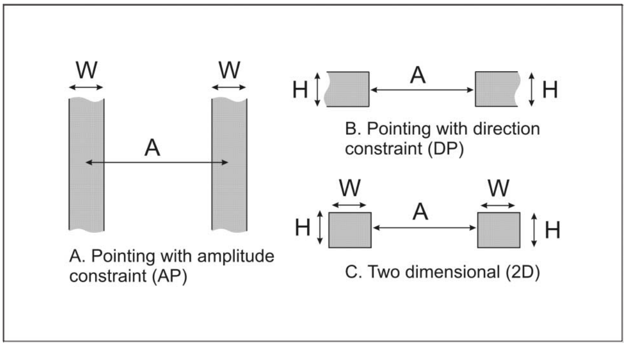


Figure 10. **Schematic representation of single- and two-dimensional Fitts' pointing paradigm: *AP* (A, left) and *DP* (B, top right) tasks, and a 2D (C, bottom right) task. Regions in gray denote the target areas. Adapted from Accot & Zhai (2003, Figure 1, p. 193, © 2003 ACM, Inc.). Adapted with permission from authors and ACM, Inc.**



pointing with direction constraints. Most targets in the applications of HCI (buttons, radio buttons, checkboxes, etc.), however, have both height and width constraints, and are two-dimensional (Figure 9).

MacKenzie and Buxton (1992) employed a 2D paradigm (Figure 10) but presented alternate quantification of W to accommodate movement amplitudes that involve an approach angle. They found that movement time was higher when the approach angle was 45° than when it was 0° or 90° (relative to the horizontal axis). It has been found that for rectangular targets, as the angle approaches 90° from 0°, the roles of the target width and height reverse (MacKenzie, 1992). (See Whisenand & Emurian, 1999, for an example of recommended guidelines on designing targets.) More recently, Fitts' Law was extended beyond the univariate or bivariate pointing (Accot & Zhai, 2003) on a flat plane to trivariate pointing using volumetric display (Grossman & Balakrishnan, 2004).

Semantic Pointing. Recall that the numerator, *A*, and denominator, *W*, of the fraction in Equation 12 can be manipulated to yield a lower ID. In other words, theoretically, a pointing task can be optimized by manipulating the targets, such that A is decreased or W is increased. Researchers have recently introduced the concept of *semantic pointing* (Blanch, Guiard, & Beaudouin-Lafon, 2004; Guiard, Blanch & Beaudouin-Lafon, 2004;) that involves both decreasing A and increasing W. Essentially, the positions of the cursor and the mouse are tracked and converted into a display-control (DC) ratio. The DC changes as a function of the cursor's proximity to targets such that there is a "manipulation of the relative sizes of objects in visual and motor space" (Blanch et al., 2004, p. 521). The resulting enhancement is a visual interface that appears to aid the user in acquiring targets, such as icons and buttons.

Text Entry on Soft Keyboards. With the ubiquity of handheld devices, such as PDAs and cellphones, and the increase in their functionality, the text entry on GUI has also been an area where Fitts' Law has been successful. A recent focus is the evaluation of the stylus tapping on soft keyboards, or a graphic representation of a computer keyboard (Soukoreff & MacKenzie, 1995). MacKenzie, Zhang, and Soukoreff (1999) evaluated six types of keyboard layouts—QWERTY, ABC, Dvorak, Fitaly, JustType, and telephone—and reported the novice and expert typing speeds predicted by a model first reported in Soukoreff and MacKenzie (1995) that was based on Fitts' Law, Hick-Hyman Law, and a linguistic frequency table. The prediction of their model and the findings in Soukoreff & MacKenzie (1995) and MacKenzie, Zhang, and Soukoreff (1999) lend support to the superiority of soft keyboards with QWERTY layout over other the forms of layout. MacKenzie et al. (1999) attribute this to the skill transfer from touch typing regular desktop keyboards, a hypothesis that was later confirmed by a study that demonstrated a moderate positive correlation between the two skills (MacKenzie & Zhang, 2001) for users who are already experienced with the QWERTY layout.

Navigation. Another area that Fitts' Law has been proven applicable is controlling navigation within a GUI environment, such as panning and zooming. One such concept is *multi-scale pointing* (Guiard, Beaudouin-Lafon,

& Mottet, 1999; Guiard, Beaudouin-Lafon, Bastin, Pasveer, & Zhai, 2004; Guiard, Bourgeois, Mottet, & Beaudouin-Lafon, 2001), which extends Fitts' paradigm beyond the 1D arena by allowing users to perform zooming and panning. Guiard et al. (1999) employed Fitts' Law to investigate a two-scale pointing and reported that the law was observed at an ID of 12.2 bits. More recently, Guiard et al. (2004) replicated this finding and additionally reported that the inverse of Fitts' Law slope is proportionate to the view size.

6. INTEGRATION OF THE LAWS

The earliest attempt to combine the Hick-Hyman Law and Fitts' Law was probably reported by Kuttan and Robinson (1970, as cited by Hoffmann & Lim, 1997). The better known work that investigated the fusion of the two laws is that of Beggs, Graham, Monk, Shaw, & Howarth (1972). To combine Hick's task and Fitts' task, Beggs et al. (1972) had participants aim for randomly indicated targets with a pencil from a home position along to clicks of a metronome. Their results showed that Fitts' Law did not hold in the fusion. Hoffmann and Lim (1997) also attempted to combine the two laws using a home-to-target paradigm but they tested their participants with both sequential tasks and concurrent tasks. In the former task, participants first react to a visual stimulus (light) and then make a movement from a home position to a target position. In the latter task, participants were required to lift their fingers from the home position before knowing where the target is. Hoffmann and Lim reported that total time taken in the sequential task was simply a sum of the decision time and movement time. However, total time taken in the concurrent task showed substantial interference, thus the fusion of Hick's Law and Fitts' Law was not entirely successful unless the combination of the two tasks is sequential to the experimental procedure.

The most recent effort to combine Hick-Hyman Law and Fitts' Law was reported by Soukoreff & MacKenzie (1995) in modeling performance in text entry using soft keyboards. Soukoreff & MacKenzie employed the two laws and Mayzner and Tresselt's (1965, as cited by Soukoreff & MacKenzie, 1995) linguistic frequency table to formulate a model that would predict a theoretical upper text entry speed for experts and a lower upper text entry speed for novices in using a stylus and a soft keyboard to enter text. MacKenzie, Zhang, and Soukoreff (1999) applied the model to evaluate soft keyboards of a variety of keyboard layouts but were unable to fit all the empirical data with their model. Sears, Jacko, Chu, and Moro (2001) submitted that using the Hick-Hyman Law to predict visual scan time was inappropriate. MacKenzie and Zhang (2001), in a study using randomized layout to simulate novice performance, would later report the incompatibility of the two laws in their model.

7. THE HICK-HYMAN LAW AND HCI

It is clear in the preceding sections that both the Hick-Hyman Law and Fitts' Law share much in common: (a) Both laws were analogies based on Shannon and Weaver's (1949) Information Theory soon after its introduction, (b) both laws employed temporal dependent measures and accuracy to address performance rates and limits of a human system, and (c) both have received substantial support in research that demonstrated their generality and in process models that explained possible underlying mechanisms. When one considers HCI research and applications of the laws, however, the Hick-Hyman Law falls short.

A possible reason why the Hick-Hyman Law lacked the momentum in HCI is related to the criticisms raised by Laming (1966, 1968) about the law's analogy to the classic Information Theory. Laming (1966) pointed out discrepancies between Shannon's original theory and Hick's analogy and later stated that "[t]he attempt to explain choice-reaction times in terms of Communication Theory must now be abandoned … it has been shown that this analogy [of humans as communication system] cannot be maintained" (1968, p. 15). However, theoretical congruence to the classic theory seems like an unlikely factor because Fitts' Law should have suffered the same demise given that the analogy of the human motor system appears less congruent to Information Theory. In Hick's paradigm, stimuli are defined events with probabilities, and residual and maximal entropies are calculated in resemblance to the classic theory. This is less intuitive in Fitts' paradigm. Mackenzie (1992), a leading proponent of Fitts' Law, conceded that the "cognitive and neuromuscular factors confound the measurement of the human channel capacity, causing tremendous variation to surface in different experiments seeking to investigate similar processes" (p. 96).

It is also reasonable to speculate that the Hick-Hyman Law fell victim to what Newell and Card (1985) referred to as the eviction of the soft sciences by the hard sciences. Card and his colleagues warned that, in the sciences, technical disciplines with a stronger quantitative caliber tend to drive out the "softer" qualitative ones, even if the latter have contributions to make. As such, it is possible that the Hick-Hyman Law was not up to par. The first piece of evidence against this argument is that the Hick-Hyman Law and Fitts' Law have comparable quantitative components but the latter did not suffer the same fate. Second, HCI has shifted its focus to include what some may refer to as soft sciences, such as sociology (Carroll, 1997; Hartson, 1998). Card et al. (1983) saw the psychology of human–computer interface as individual psychology (p. 14) but today cooperative activity paradigms seem to have replaced user performance models in emphasis in HCI (Carroll, 1997; Hartson, 1998).

7.1. Difficulty in Application

The most obvious obstacle for the Hick-Hyman Law in HCI is the difficulty in application (Stuart Card, electronic mail, October 22, 2003). To apply the law in the traditional fashion, it is necessary to first codify equivalent events involved into alternatives. The probabilities of these alternatives must then be determined to calculate their entropy (Landauer & Nachbar, 1985). One reason few HCI research projects have hardly been past this stage is because there was no need to engage in the complexity of the information theoretic measures. When a task can be viewed in terms of alternatives and quantified in bits, it is likely to be too simplistic to be practical and useful. This inherent difficulty most likely led to an absence of proponents to champion its application in the field and a resistance to apply information theoretic measures on the grounds of parsimony (George Miller, electronic mail, December 31, 2003). The attentive reader would have noticed the several mentions of MacKenzie in conjunction with Fitts' Law. I. S. MacKenzie wrote his doctoral dissertation (MacKenzie, 1991) on Fitts' Law and has since been very prolific on the topic.

7.2. Complexity of Stimuli

A related problem is the stimuli involved in HCI. The Hick-Hyman Law has considerable progress outside of HCI, such as in experimental psychology and psychometrics, due in part to the fact that such studies necessarily employ simple unidimensional stimuli (such as conspicuous lights, simple shapes or symbols, etc.) to reduce confounding. Because contemporary interfaces in HCI involve highly complex interfaces (such as buttons, menus, text, animation, etc.) that frequently comprise a variety of multidimensional stimuli, deriving the informational content is more challenging. For example, in conventional Hypertext Markup Language (HTML), one can code a text to appear in a large combination of font size, colors, and typeface, in addition to style, such as bold, italicize, underlined, and so on. The stimuli involved in Fitts' Law have also matured in complexity over the years, evolving from traditional unidimensional targets to contemporary multidimensional targets, but its progress has been relatively gradual.

7.3. Levels and Types of Performance

Another distinction between the Hick-Hyman Law and Fitts' Law is the level and type of user performance each captures. Fitts (1954) employed somewhat monotonous tasks (tapping to metal strips with a stylus, transferring disks to pins, and transferring pins to holes) and thus captured a type of human performance that was relatively more automated, kinesthetic, and related to dexterity. As such, the law was immediately applicable to highly familiar or learned tasks, such as typing on a QWERTY keyboard or maneuvering the mouse to click on a button on a GUI. Hick and Hyman, on the other hand, incorporated degrees of unpredictability in their stimuli that required the participants to remember the correct responses. Such performance, as highlighted earlier, can gradually improve over time, such as with optimal SRC and practice. As such, the Hick-Hyman Law appears to be optimal in predicting novice performance in tasks that engage users in some level of cognitive process.

8. CONCLUSION

The validity, theoretic roots, and quantitative caliber of the law seem unlikely to be significant factors that prevented the Hick-Hyman Law from gaining momentum in HCI. What is plausible is a combination of several factors. First, the inherent difficulty in applying the law is obvious. This is likely due to the relatively more complex stimuli in HCI. Additionally, an inability to account for performance in a highly familiar, automated, or trained task may have also limited the law's applicability to HCI problems. Nevertheless, within limits, the relationship between information load and choice-reaction time captured by the Hick-Hyman Law is robust and demonstrable at the basic level. The challenge lies in codifying complex multidimensional stimuli and extending the law beyond novel performance.

NOTES

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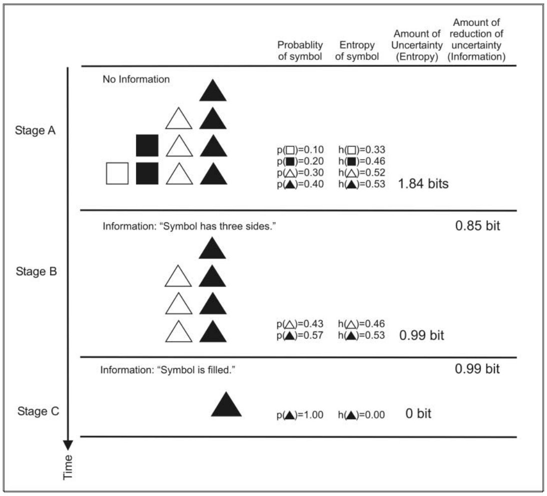
APPENDIX A. CALCULATING ENTROPY

This appendix provides a primer on the concept of entropy and how it is calculated from a set of events with defined probabilities. Of interest is the comparison between the entropy of a set of equiprobable events and entropy of a set of nonequiprobable events.

Suppose one is to determine which of four types of symbols is a preselected but unknown symbol (Figure 10). With the assumption that all symbols have equal chances of being the right answer, the amount of uncertainty about the right answer in this hypothetical scenario is quantified as $\log_2 4 = 2$, or 2 bits. This amount of uncertainty is called the entropy. In other words, it indicates the potential of information the four symbols carry.

Suppose a clue is given that the correct symbol is a triangle, the square symbols may be eliminated. Thus the uncertainty now is $\log_2 2 = 1$. As such, it can be deduced that the information about the correct symbol having three sides carries 1 bit of information. If it were further given that the symbol is filled, another 1 bit of information would be available and the uncertainty would be reduced to 0 bits because the answer has been determined. When alternatives are equiprobable, information or uncertainty is quantified as:

***Figure A-1.* Reduction of uncertainty in a "rigged" set of alternatives. In this example, not all symbols have equal probabilities. The open square has a probability of 0.1 of being the correct symbol, filled squares 0.2, open triangles 0.3, and filled triangles 0.4. This information content of the first clue that the symbol has four sides is calculated by subtracting the information contents of the remaining uncertainty (0.99 bit) from the starting amount of entropy (1.84 bits).**



$$H = \log_2 N \tag{14}$$

where H is the quantity of information or uncertainty, and N is the number of equiprobable alternatives.

It must be emphasized that Equation 14 is only applicable when the alternatives are equiprobable. Let's suppose that the probability for each symbol is different and "rigged" as such: 0.1, 0.2, 0.3, and 0.4. Equation 16 must be modified to take the unequal probabilities into account. First, the amount of information for each alternative is calculated as a weighted average of the probabilities of the entire set:

$$h_i = p_i \log_2 \left(\frac{1}{p_i} \right) \tag{15}$$

where h_i is the amount of information carried by event i, with a probability of p_i of occurring in the set. In the forementioned example, each h_i would be calculated individually as such:

$$h_1 = 0.1 \left[\log_2 \left(\frac{1}{0.1} \right) \right] = 0.33 bit \tag{16}$$

$$h_2 = 0.2 \left[\log_2 \left(\frac{1}{0.2} \right) \right] = 0.46 bit \tag{17}$$

$$h_3 = 0.3 \left[\log_2 \left(\frac{1}{0.3} \right) \right] = 0.52 bit \tag{18}$$

$$h_4 = 0.4 \left[\log_2 \left(\frac{1}{0.4} \right) \right] = 0.53 bit \tag{19}$$

All the h's are then summed:

$$H = \sum_{i=1}^{n} p_i \log_2 \left(\frac{1}{p_i} \right) \tag{1, repeated}$$

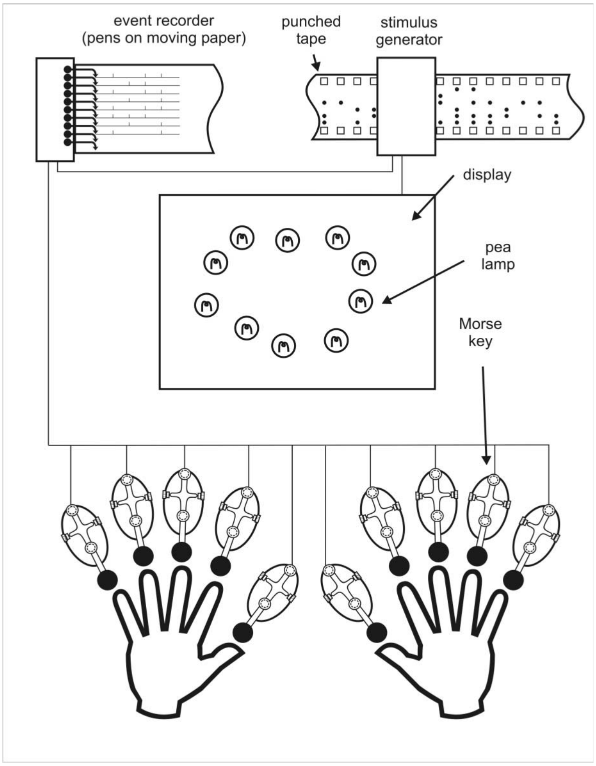
where n is the number of alternatives. This yields 1.84 bits in the "rigged" example.

Equation 1 is the formula for quantifying uncertainty or information with both equal and unequal probabilities. Amount of information that was faithfully transmitted is typically designated H_T. Note that there is a 0.16 bit reduction from the first equiprobable set to the "rigged" set. Contrasting the two demonstrates that the amount of information or uncertainty is maximal when all elements are equiprobable.

APPENDIX B. AUTHOR'S IMPRESSION OF THE APPARATUS IN HICK (1952)

Holes with 1/8" diameter were punched on 35 mm film with driving perforation (holes that run alongside the film). The width of the film allows for six holes, aligned to each perforation, to be punched. The film is driven through a stimulus, one perforation at a time, and is passed through six pairs upper and lower contacts. When there is a hole at a particular position on the film, the corresponding pair of upper and lower contacts will close a switch. Using the combination of these six switches, Hick's device could generate up to 2^6 or 64 possible alternatives (Hick, 1951), but only four were needed to code his experiments.

Figure B-1. **Author's impression of the apparatus in Hick (1952). The stimulus generator is described in Hick (1951). The others are based on information provided in Hick (1952).**



APPENDIX C. AUTHOR'S IMPRESSION OF THE APPARATUS IN HYMAN (1953)

Figure C-1 illustrates the author's impression of Hyman's apparatus using information provided in Hyman (1953) and through personal electronic communication with R. Hyman (August 2004).

Figure C-1. **Author's impression of the apparatus in Hyman (1953) based on information given in Hyman (1953) and information provided by R. Hyman through personal electronic communication.**